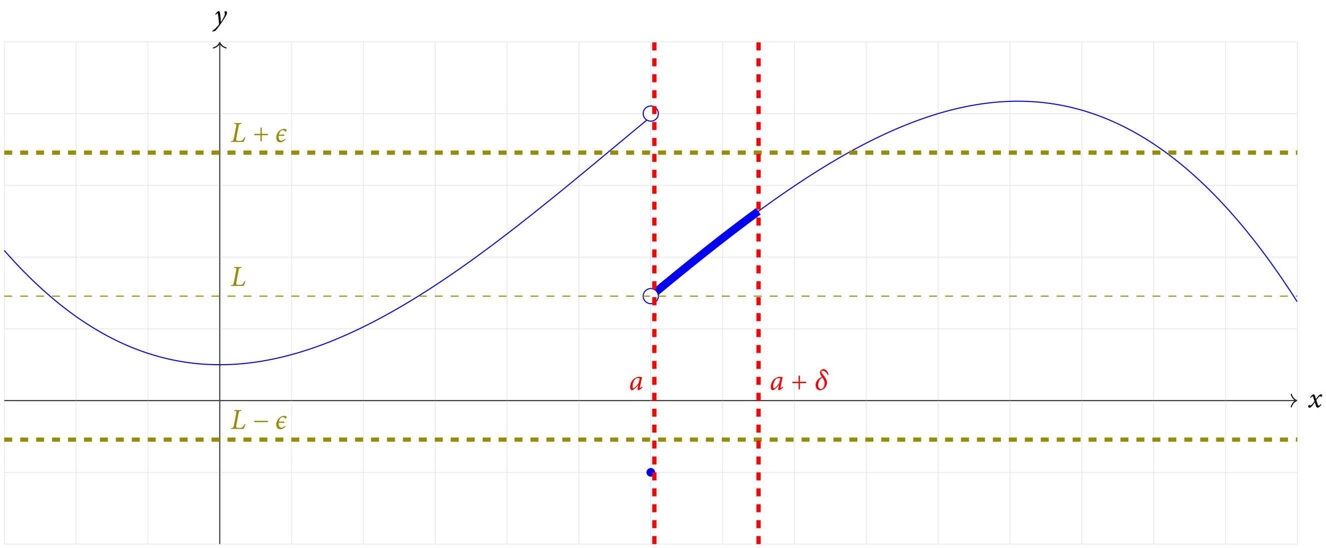
Math 1A: Calculus **Instructor: Alexander Paulin**

Handout: The Precise Definition of a Limit

Discussions 201, 203 // 2018-09-14

The textbook has pictures illustrating the precise definition of a two-sided limit. Here are pictures for one-sided limits. The first one is for a *right-hand* limit:

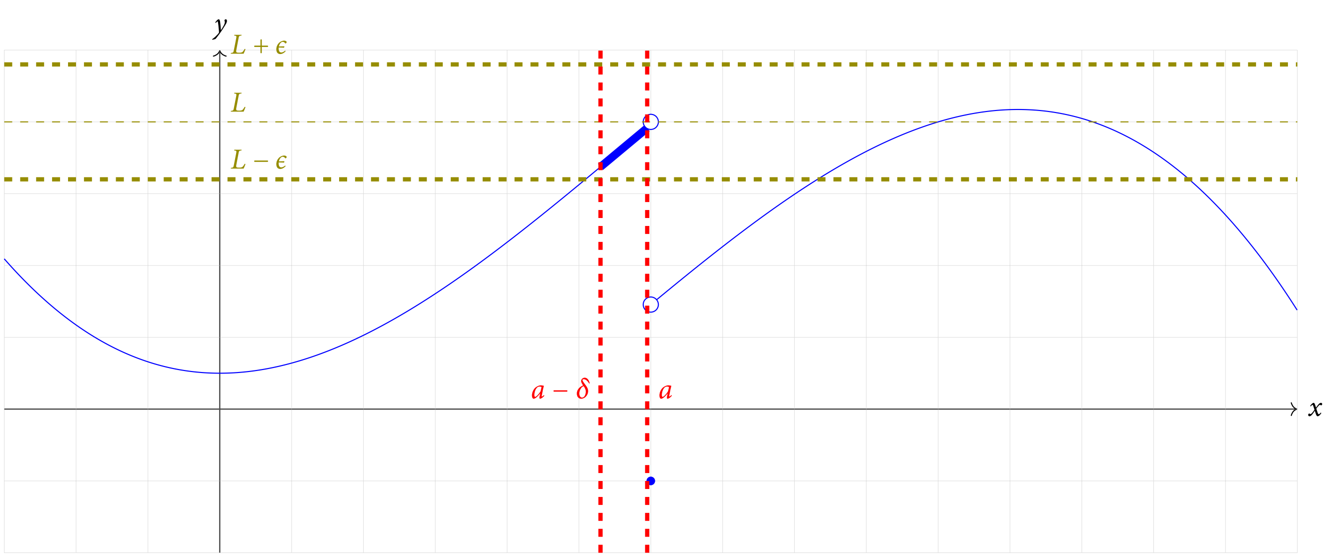


We say that $\lim_{x\to a^+} f(x) = L$ if, for *each* $\epsilon > 0$, it is possible for us to find $\delta > 0$ such that

$$a < x < a + \delta \implies |f(x) - L| < \epsilon.$$

In other words, for every horizontal strip *around* $y = L$, we can find a vertical strip immediately to the *right* of $x = a$ such that the graph of $y = f(x)$ in that vertical strip is contained within the horizontal strip.

Next, a *left-hand* limit:



We say that $\lim_{x\to a^-} f(x) = L$ if, for *each* $\epsilon > 0$, it is possible for us to find $\delta > 0$ such that

$$a - \delta < x < a \implies |f(x) - L| < \epsilon.$$

In other words, for every horizontal strip *around* $y = L$, we can find a vertical strip immediately to the *left* of $x = a$ such that the graph of $y = f(x)$ in that vertical strip is contained within the horizontal strip.

Problem 1. Using the definitions, prove that $\lim_{x\to a} f(x) = L \iff \lim_{x\to a^-} f(x) = \lim_{x\to a^+} f(x) = L$.

Continuity

Recall that we say a function is *continuous at a* if

$$\lim_{x\to a} f(x) = f(a).$$

Problem 2. Show that $f(x) = x$ is continuous everwhere.

If you believe the various limit laws that we've covered so far, from the preceding problem you can show the following.

Problem 3. Show that polynomials are continuous everywhere. (Hint: product and sum laws for limits.)

That being said, sometimes you might be asked to verify a limit directly using the limit laws, so you should know how to do that:

Problem 4. Show that $\lim_{x\to 3}(x^2 + x - 4) = 8$ using the definition of a limit.